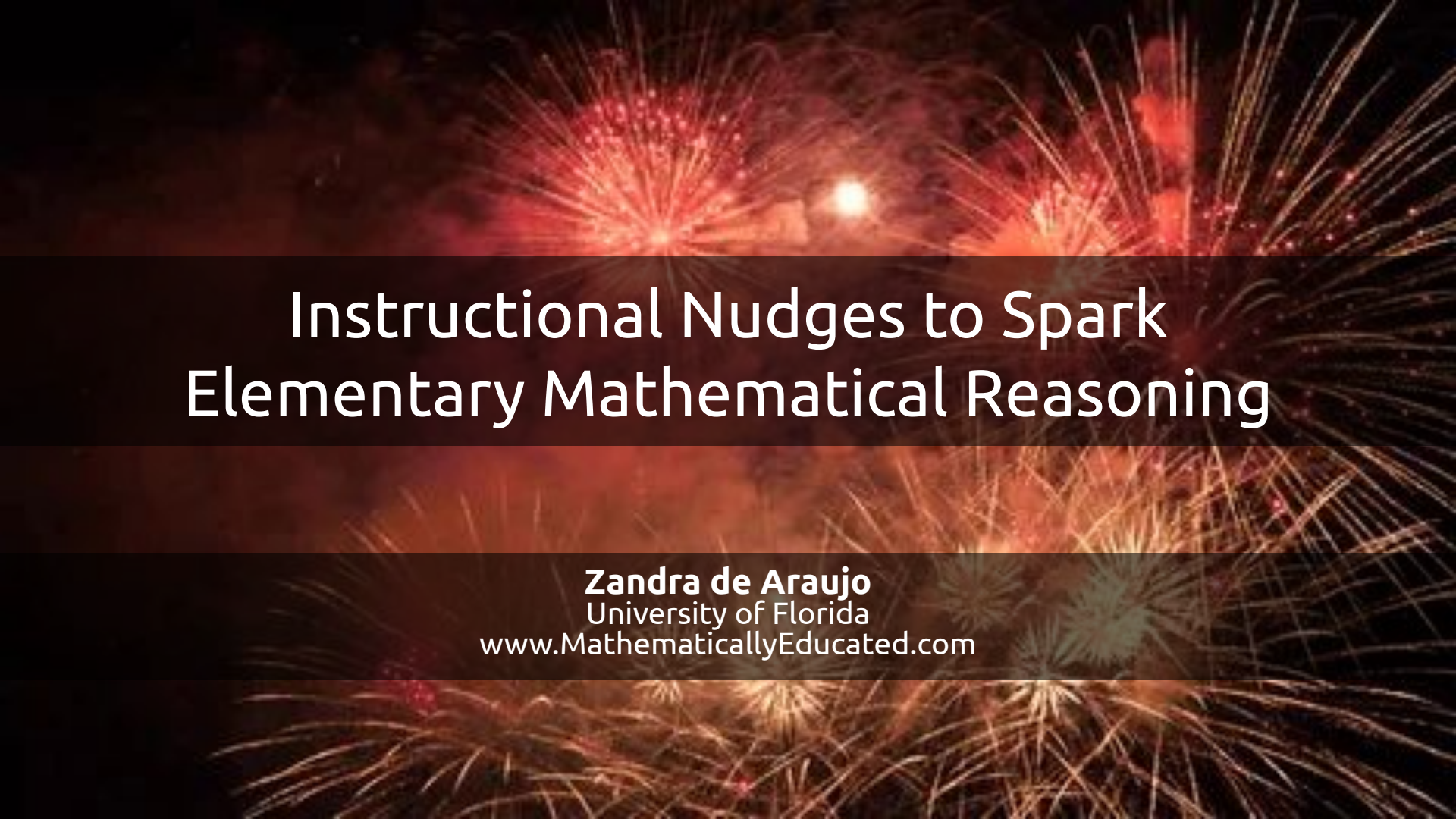




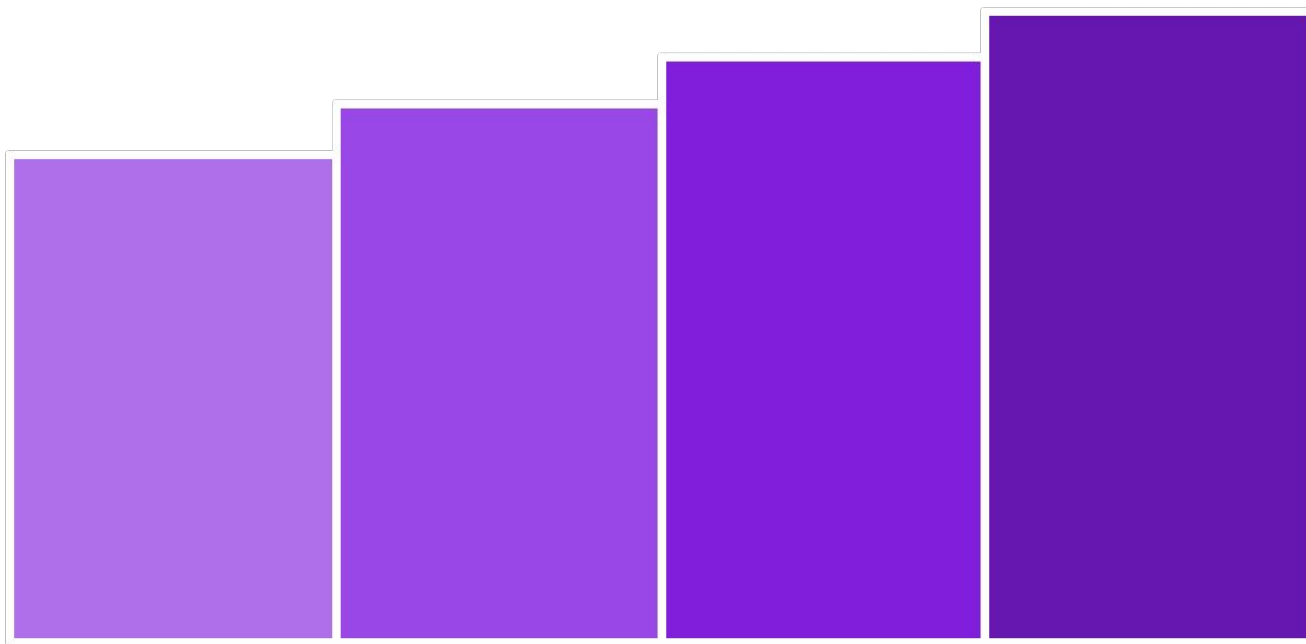
Instructional Nudges to Spark Elementary Mathematical Reasoning

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What is reasoning?

**How do we know when students
are reasoning?**

**Why aren't students reasoning in
the mathematics classroom?**



Practice-Driven PD

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Olumide Banjo



The views expressed here do not necessarily reflect those of the National Science Foundation.

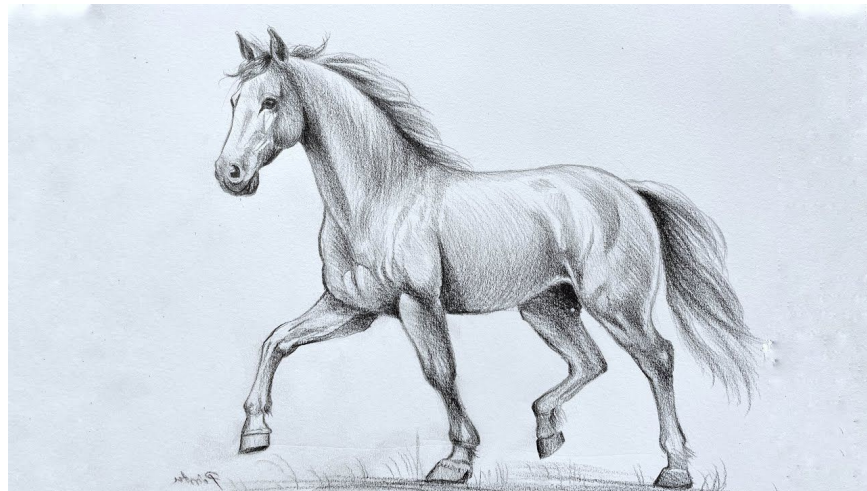


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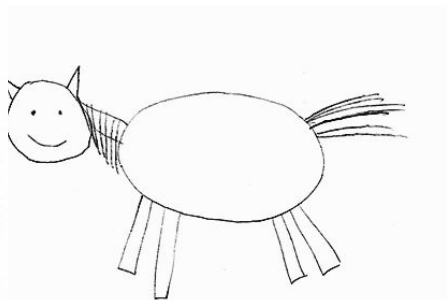


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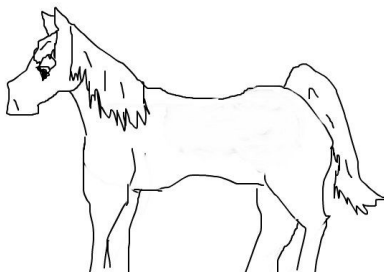
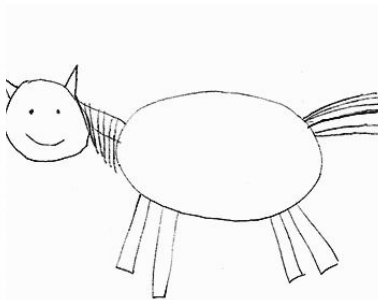


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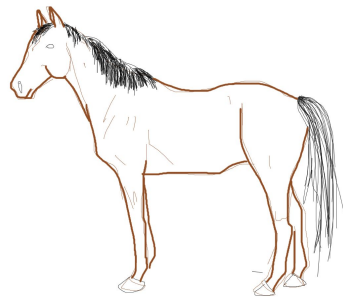
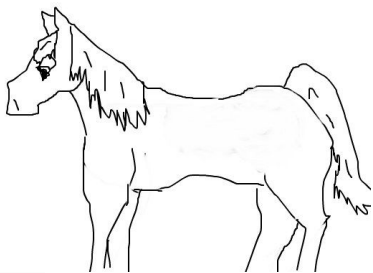
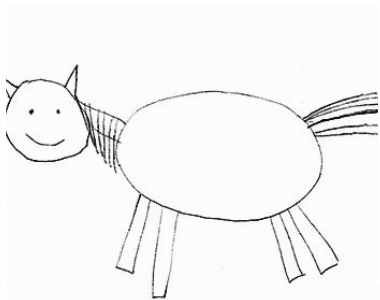


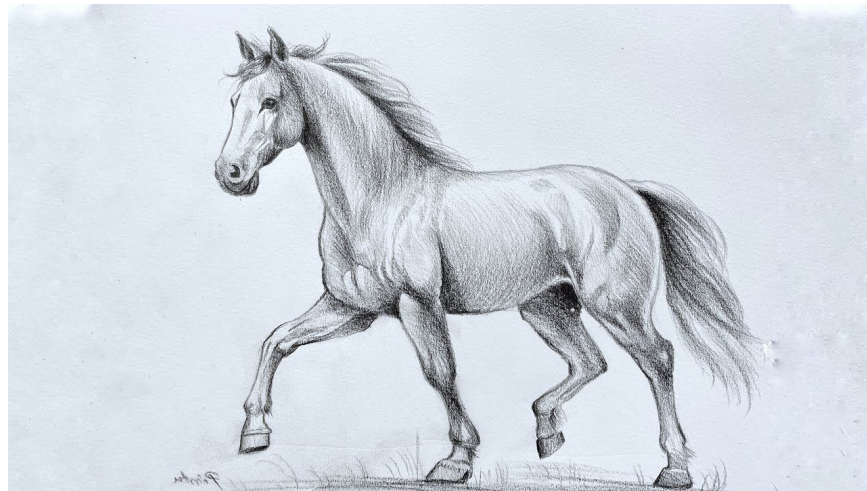
GOAL



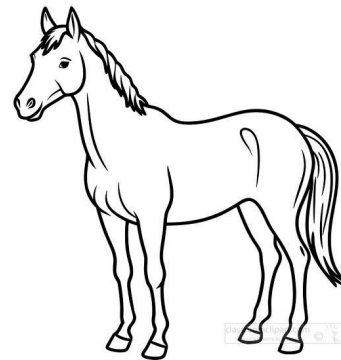
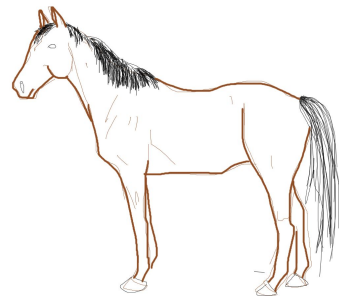
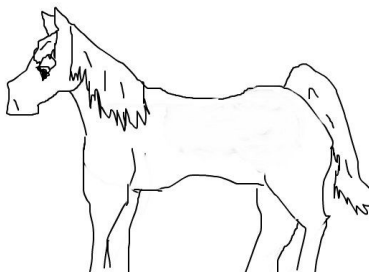
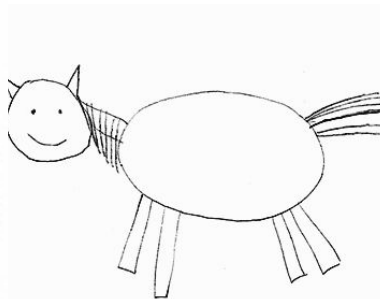


GOAL





GOAL



Instructional Nudges

Modest suggestion intended to be quickly and easily incorporated into teachers' current practice.



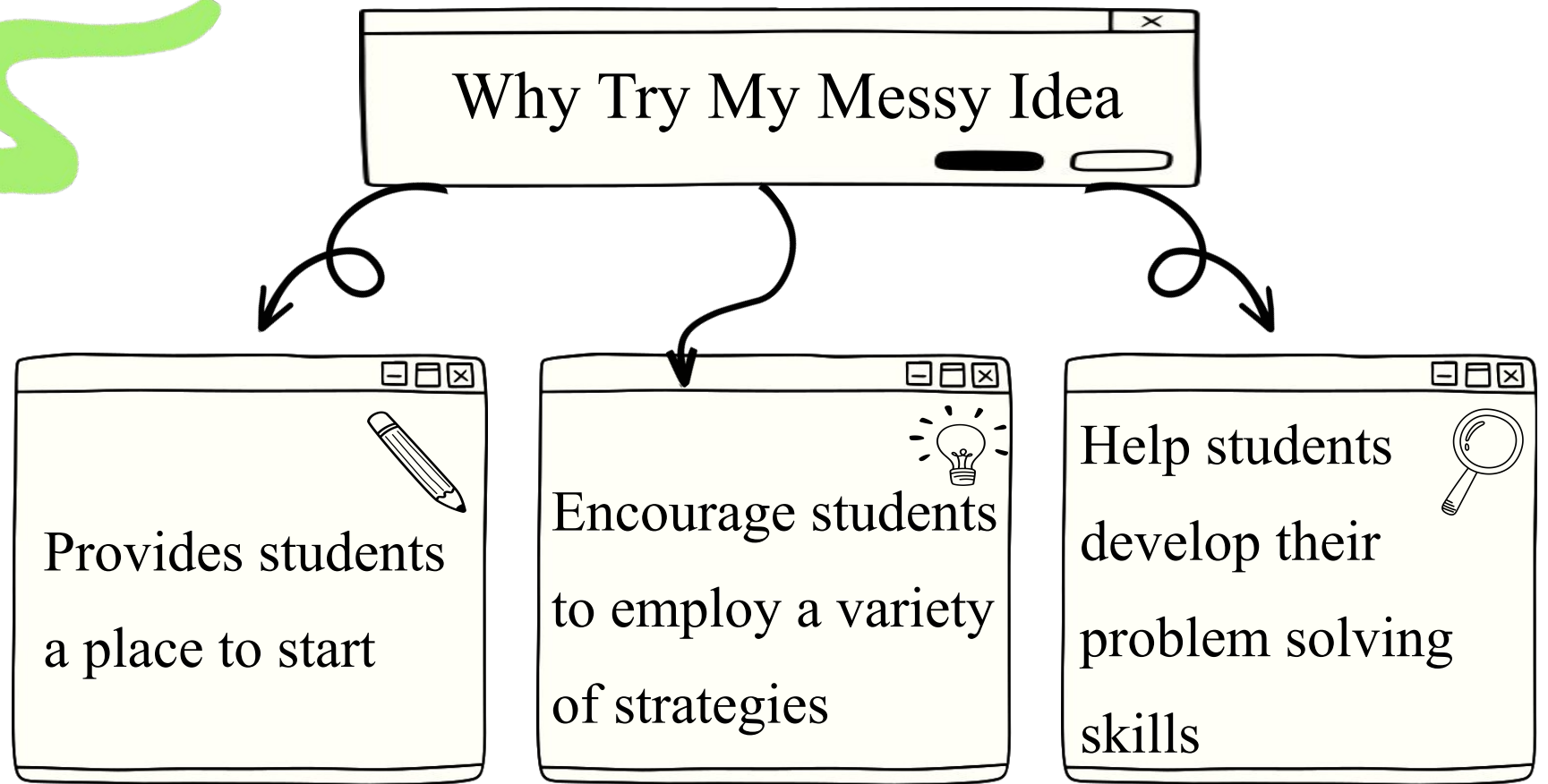
My Messy Idea

What is My Messy Idea

- 01 It is an instructional strategy where students jot down their initial thoughts or ideas about the problem before solving it
- 02 Initial Ideas could take any form
- 03 Not to be graded



Why Try My Messy Idea



```
graph TD; A[Why Try My Messy Idea] --> B[Provides students a place to start]; A --> C[Encourage students to employ a variety of strategies]; A --> D[Help students develop their problem solving skills];
```



Provides students
a place to start



Encourage students
to employ a variety
of strategies



Help students
develop their
problem solving
skills



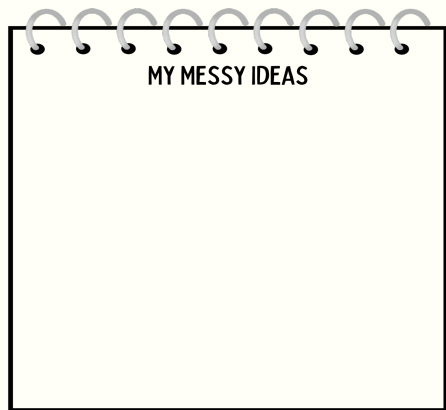
Example of Messy Idea Activity



Imagine writing down every possible four-digit number using the digits 1, 2, 3, and 4. What would be the sum of all these numbers?

Problem adapted from mathschallenge.net

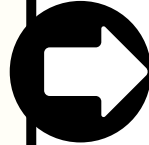
Share your initial thoughts in the messy idea box below.



MY MESSY IDEAS

...

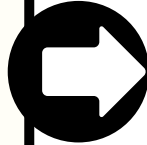
Students Initial thoughts
or ideas could include



1	2	3	4
1	3	2	4
1	3	4	2
1	2	4	3
2			
1	4	3	2
1	4	2	3

If 1 appears 6 times in the
One's place, then that could be
true for the other digits.

Students' messy ideas
could include various
forms



Messy Idea

Create a list of possible places
the digits could fit.

1	2	3	4
1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 4 1	3 2 1 4	4 2 3 1
1 3 4 2	2 3 1 4	3 2 4 1	4 2 1 3
1 4 3 2	2 4 1 3	3 4 2 1	4 3 1 2
1 4 2 3	2 4 3 1	3 4 1 2	4 3 2 1
add? 7 9	9 8	13 7 7 6	2 5 3 3 2

6 is in one's place

6 2's in tenth place

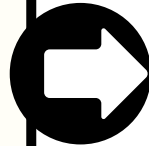
11 3's in hundred place

11 4's in thousandth place

} Then what?
back to table.



Students' messy ideas
could include various
forms



My Messy Idea

1, 2, 3, 4

How many times can I arrange these numbers? Do they need to repeat??

1	2	3	4
2	1	4	3
3	4	2	1
4	3	1	2

{	1 2 3 4	1 2 3 4	—	}
	1 2 4 3	1 3 2 4	—	
	1 3 2 4	1 4 3 2	—	
	1 3 4 2	1 3 4 2	—	
	1 4 2 3	1 4 2 3	—	
	1 4 3 2	1 2 4 3	—	

↓ ones ↓ tenth ↓ hundredth ↗ thousands

There are 6 possible
place value for
each digit

6 - 1
6 - 2
6 - 3
6 - 4

$6 \times 4 = 24$
↓ ↓
places four digits

or

$6(1+2+3+4)$
 $6(1)(1+2+3+4)$

$6(1) \rightarrow$
 $6(2) \rightarrow$
 $6(3) \rightarrow$
 $6(4) \rightarrow$

$6(10)^3(1+2+3+4)$
 $6(10)^2(1+2+3+4)$
 $6(10)^1(1+2+3+4)$
 $6(10)^0(1+2+3+4)$

To Spark Mathematical Reasoning
Encourage Students to **Analyze Solutions**

To Spark Mathematical Reasoning
Encourage Students to **Analyze Solutions**

Rate & Review



Johannes · June 15, 2018



Verified Purchase

Size: Original version

It is definitely a bucket

Includes all the features you love from a bucket:

- Roundness
- Bucket-ness
- Comfort plastic grip
- Holds water and other things

Easy 5 stars.



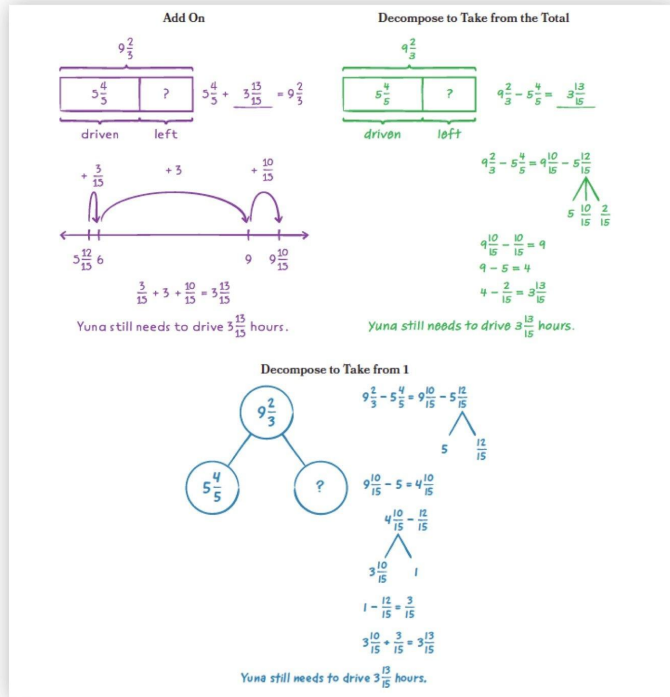
One Star

By Joe Watson - December 14, 2014

There were no wolves in the movie.

0 of 3 people found this review helpful

Worked Examples / Error Analysis



Error Analysis - Example

Where's the Mistake?

Question: $72.6 \times 28.3 =$

Find the Error:

$$\begin{array}{r} 72.6 \\ \times 28.3 \\ \hline 2168 \\ 58080 \\ + 145200 \\ \hline 20544.8 \end{array}$$

Model the Correct Solution:

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A $\frac{4}{5} \div \frac{2}{10}$

$$\frac{8}{10} \div \frac{2}{10}$$

$$\frac{8 \div 2}{10 \div 10} = \frac{4}{1} = 4$$

B $\frac{4}{5} \div \frac{2}{10}$

$$\frac{4}{5} \div \frac{1}{5}$$

Duh, it's 4!

C $\frac{4}{5} \div \frac{2}{10} = \frac{4}{5} \times \frac{10}{2} = \frac{40}{10} = 4$

Rate & Review



Rate these solutions and write a brief review (why did you give that rating?)



$$5 \overline{) 655}$$

Alpha

$6 \div 5 = 1$ so I put 1 in the hundreds spot and $6 - 5 = 1$ so I bring 1 and the first 5 down. $15 \div 5 = 3$ so I put 3 in the tens spot. Then finally $5 \div 5 = 1$ so it's 131.

$$5 \overline{) 655}$$

Beta

$$5 \times 100 = 500$$

$$5 \times 20 = 100$$

$$5 \times 11 = 55$$

131

Rate and Review

Students might rate based on...

- Correctness
- Clarity
- Style, Elegance
- Some unique combination thereof
- What else?

Rate and Review

Students might rate based on...

- Correctness
- Clarity
- Style, Elegance
- Some unique combination thereof
- What else?



CAUTION

Your actual students' work should NOT be used for Rate and Review in order to avoid student embarrassment or anxiety.

To Spark Mathematical Reasoning
Encourage Students to **Analyze Solutions**

Rate & Rewrite

There are 36 students going on a field trip. Each van can hold 8 students. How many vans are needed?

Solution:

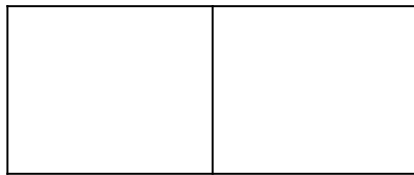
I know that $8 + 8 + 8 + 8 = 32$, so 4 vans can hold 32 students.

There are $36 - 32 = 4$ students left. Instead of getting another whole van, I split the last van into halves because only 4 students are left.

Since 4 is half of 8, I need $\frac{1}{2}$ of a van.

So I need $4 \text{ vans} + \frac{1}{2} \text{ van} = 4\frac{1}{2} \text{ vans}$

A shape is made by putting two 4-inch squares side by side. What is the perimeter of the new shape?



Solution:

I drew the two squares and counted all the sides. Each square has 4 sides, so that is 8 sides total. Each side is 4 inches long. $8 \times 4 = 32$

But the two squares are touching in the middle, so I crossed out one 4-inch side. $32 - 4 = 28$

So the perimeter is 28 inches.

Subtract $537 - 503$

Solution:

I know that subtraction is how far apart they are. I wanted to make an easier problem so I moved the 537 up 3 and the 503 down 3 to get $540 - 500$ which was easier to figure out that the answer is 60.

To Spark Mathematical Reasoning
Encourage Students to **Make Connections**

To Spark Mathematical Reasoning
Encourage Students to **Make Connections**

Pairing Problems



PAIRING PROBLEMS

Sometimes two problems are better than one



Typically students experience...

- Problems one at a time
- Mathematics as disconnected ideas

1. $150 - 70$

5. $978 - 897$

2. $510 - 430$

6. $787 - 9$

3. $999 - 680$

7. $86 - 6$

4. $897 - 798$

8. $500 - 420$

**Which 2 problems do you think are most different?
How are they different?**

A) $1.25 + 5.75$

F) $0.8 + 9.2$

B) $2 + 1 + 4$

G) $521 + 755$

C) $575 + 125$

H) $7 + 3 + 2$

D) $6.5 + 0.5$

I) $3.5 + 2.8 + 5.7$

E) $4.45 + 6.55$

J) $12.5 + 0.575$

**Which 2 problems do you think are most similar?
How are they similar?**

A. $32 + 18$

D. $35 + 38$

B. $47 + 26$

E. $19 + 24$

C. $53 + 39$

F. $17 + 44$

**Create another problem that would fit in this set.
Explain your thinking.**

1. Mia has 18 stickers and Noah has 11 stickers; how many more stickers does Mia have than Noah?
2. A red tree has 23 apples and a green tree has 17 apples; how many more apples does the red tree have?
3. Emma read 14 pages and Lucas read 9 pages; how many more pages did Emma read?
4. One fish tank has 26 fish and another fish tank has 19 fish; how many more fish are in the first tank?

Create another problem that would NOT fit in this set. Explain your thinking.

Pairing Problems

Students might pair based on...

- Problem surface features
- Problem structural features
- Aspects of the solutions produced
- Nothing much (“smart aleck pairing”)
- What else?

To Spark Mathematical Reasoning
Encourage Students to **Make Connections**

Four the Win

Four the Win

Idea #1

Idea #2

Idea #1 in a
different format

BLANK

Four the Win!

A)

$$2x = y$$

B)

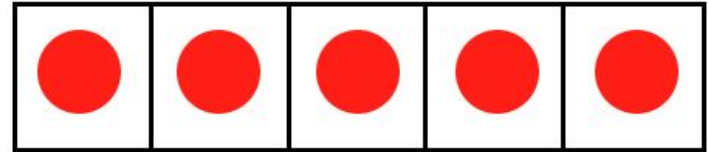
$$2y = x$$

C)

D)

The value of y is twice as big
as the value of x .

Four the Win



Four the Win!

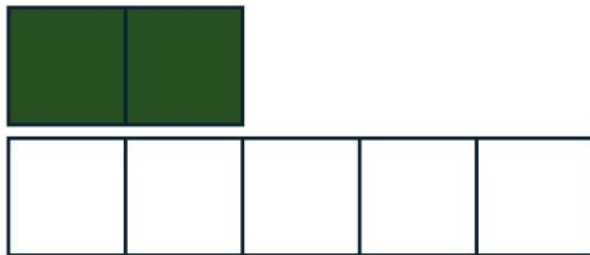
A)

There are 2 weekend days (Saturday, Sunday) for every 5 weekdays (Monday–Friday).

B)



C)



D)



Four the Win!

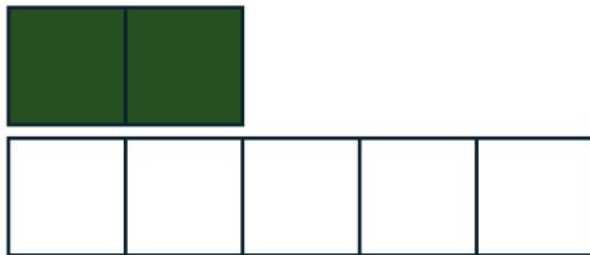
A)

There are 2 weekend days (Saturday, Sunday) for every 5 weekdays (Monday–Friday).

B)



C)



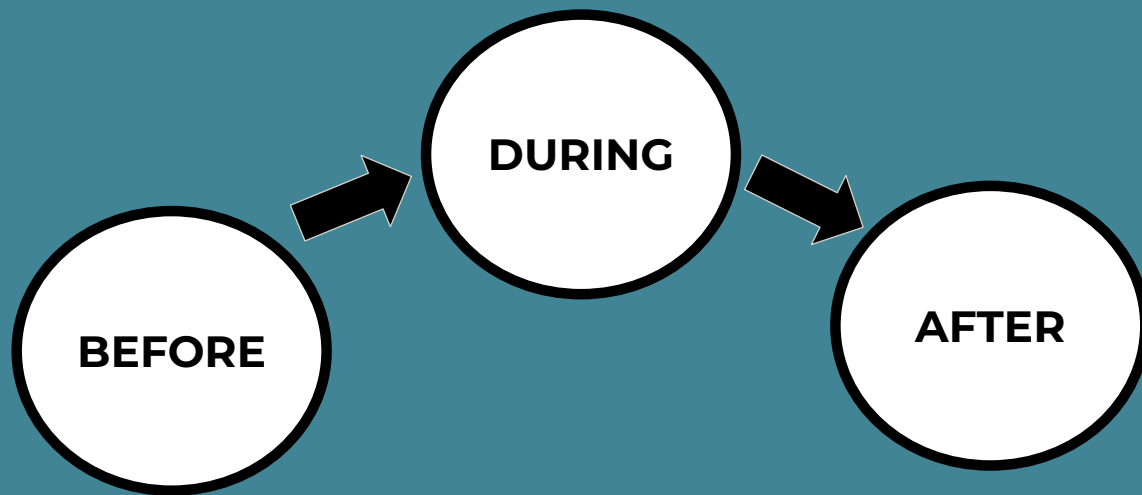
D)

- “I have cereal for breakfast 2 out of every 5 weekdays.”
- “Two out of every five pets is a cat,”
- “There are 2 green blocks for every 3 white blocks.”

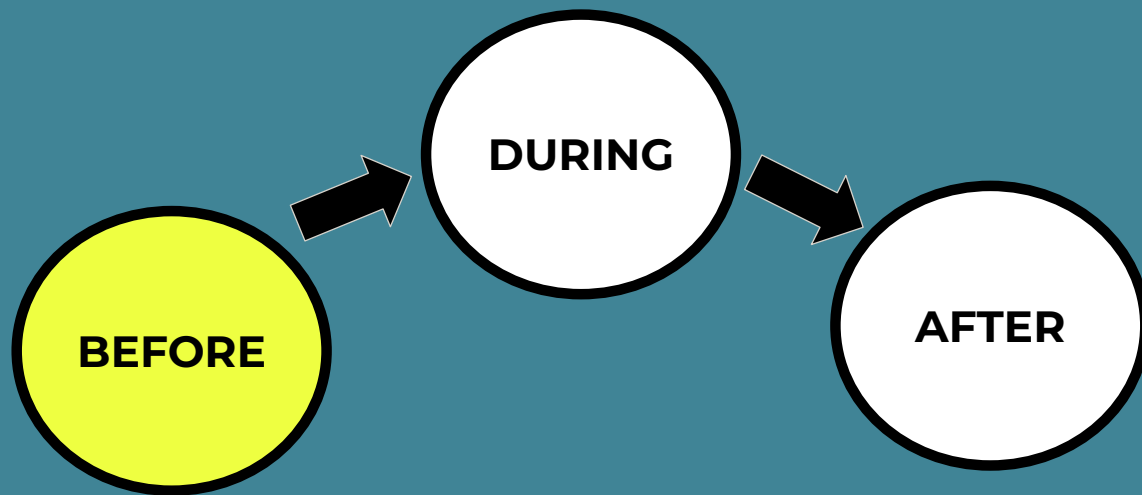
To Spark Mathematical Reasoning
Make the Most of the Launch

To Spark Mathematical Reasoning

Make the Most of the Launch



To Spark Mathematical Reasoning **Make the Most of the Launch**



To Spark Mathematical Reasoning
Make the Most of the Launch

And the Answer is

Rosa has 12 cups of condensed milk for her grandmother to make many batches of tres leches cake. So far, her grandmother has used 8 and $\frac{1}{4}$ cups of condensed milk. How much condensed milk does she have remaining?

What is the form and format of the answer?

A pet store is preparing adoption kits for a school event.

Each adoption kit includes:

- 2 pounds of pet food
- 3 feet of ribbon
- 4 toy mice

The store has 24 pounds of pet food, 45 feet of ribbon, and 70 toy mice.

What is the greatest number of complete adoption kits the store can make? Explain your thinking.

What is the form and format of the answer?

To Spark Mathematical Reasoning
Make the Most of the Launch

Wrong Two Ways

$$5 \times \frac{2}{3}$$

AN ANSWER THAT IS TOO LARGE. HOW DO YOU KNOW?

AN ANSWER THAT IS TOO SMALL. HOW DO YOU KNOW?

THE QUOTIENT OF 653 AND 88

AN ANSWER THAT IS TOO LARGE. HOW DO YOU KNOW?

AN ANSWER THAT IS TOO SMALL. HOW DO YOU KNOW?



Thank You



Zandra de Araujo